# Comment on the deformation of quantum mechanics



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COMMENT

# Comment on the deformation of quantum mechanics

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**Abstract.** In this comment we point out a serious mistake in a recent paper by Li and Sheng and present the correct version for the $q$-deformed quantum mechanics.

Recently Li and Sheng [1] presented the one-dimensional $q$-deformed Schrödinger equation for the real deformation parameter $q$. In this comment we point out a serious mistake in [1]. Moreover we recommend the correct way to obtain the $q$-deformed Schrödinger equation by using the generalized deformed algbera [2] presented by us.

In [2] we considered the generalized deformed algebra

$$aa^+ - qa^+a = q^{\alpha N+\beta} \tag{1}$$

where we restricted the analysis to the case of $\beta = 0$ and $\alpha \neq 1$. Then the relation between the number operator and mode operator is given by the generalized deformed $q$-number

$$a^+a = [N] = \frac{q^{\alpha N} - q^N}{q^\alpha - q}. \tag{2}$$

The new $q$-derivative is defined as

$$Df(x) = \frac{f(q^\alpha x) - f(qx)}{x(q^\alpha - q)} \tag{3}$$

which satisfies the following deformed Leibniz rule

$$\begin{aligned} D(f(x)g(x)) &= f(q^\alpha x)Dg(x) + Df(x)g(qx) \\ &= Df(x)g(q^\alpha x) + f(qx)Dg(x). \end{aligned} \tag{4}$$

Letting $f(x) = x$ in equation (4) leads to

$$\begin{aligned} Dx - q^\alpha xD &= \hat{q} \\ Dx - qxD &= \hat{q}^\alpha \end{aligned} \tag{5}$$

where $\hat{q}$ is defined as

$$\hat{q}f(x) = f(qx). \tag{6}$$

Similarly, we obtain

$$\hat{q}D = q^{-1}D\hat{q}. \tag{7}$$

Then the $q$-integral is easily defined as

$$\begin{aligned} \int_0^x f(x)Dx &= (q^\alpha - q)\Sigma_{n=0}^{\infty} q^{(1-\alpha)n-\alpha} x f(q^{(1-\alpha)n-\alpha}x) \\ \int_0^\infty f(x)Dx &= (q^\alpha - q)\Sigma_{n=-\infty}^{\infty} q^{(1-\alpha)n-\alpha} f(q^{(1-\alpha)n-\alpha}) \end{aligned} \tag{8}$$

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$$\int_{-\infty}^{\infty} f(x)Dx = (q^{\alpha} - q)\Sigma_{n=-\infty}^{\infty} q^{(1-\alpha)n-\alpha} f(q^{(1-\alpha)n-\alpha})$$
$$-(q^{\alpha} - q)\Sigma_{n=-\infty}^{\infty} q^{(1-\alpha)n-\alpha} f(-q^{(1-\alpha)n-\alpha}). \tag{9}$$

Then the $q$-integral satisfies the following properties†

$$\int_{-\infty}^{\infty} f(x)Dx = \int_{-\infty}^{\infty} q^{(1-\alpha)l}\hat{q}^{(1-\alpha)l} f(x)Dx \quad \text{for any integer } l. \tag{10}$$

From the fact that

$$\int_{-\infty}^{\infty} D(\Psi^*(x)\Phi(x))Dx = 0 \tag{11}$$

where $\Phi(x)$ denotes a wavefunction and $\Psi^*(x)$ denotes a complex conjugate of the wavefunction $\Psi(x)$, we have

$$\int_{-\infty}^{\infty} D\Psi^*\hat{q}\Phi Dx = -\int_{-\infty}^{\infty} (\hat{q}^{\alpha}\Psi^*)D\Phi Dx. \tag{12}$$

Using the property (10), equation (12) reduces to

$$q^{(1-\alpha)l}\int_{-\infty}^{\infty} (\hat{q}^{(1-\alpha)l} D\Psi^*)\hat{q}^{(1-\alpha)l+1}\Psi\, Dx$$
$$= -q^{(1-\alpha)l'}\int_{-\infty}^{\infty} (\hat{q}^{(1-\alpha)l'+\alpha}\Psi^*)(\hat{q}^{(1-\alpha)l'} D\Psi)\, Dx \text{ (for any integer } l,\ l' \text{)}. \tag{13}$$

In order for equation (13) to give the Hermitian relation of $q$-derivative $D$, it should satisfy the following conditions;

$$\begin{aligned}(1-\alpha)l + 1 &= 0\\ (1-\alpha)l' + \alpha &= 0.\end{aligned} \tag{14}$$

Solving the above two equations, we have

$$\alpha = \frac{l+1}{l} = \frac{l'}{l'-1} \tag{15}$$

which gives the relation between $l$ and $l'$:

$$l' = l + 1.$$

The allowed values of $\alpha$ are given by the following string‡,

$$\alpha = \frac{p+1}{p} \qquad (p \neq 0 \text{ integer}).$$

For the special case of $\alpha$ an integer, we have two solutions:

$$\alpha = 2, l = 1, l' = 2 \quad \text{and} \quad \alpha = 0, l = -1, l' = 0. \tag{16}$$

For allowed $\alpha$'s, we have the following relation:

$$(q^{(1-\alpha)l}\hat{q}^{(1-\alpha)l}D)^{+} = -q^{(1-\alpha)l'}\hat{q}^{(1-\alpha)l'}D. \tag{17}$$

Using the fact that

$$\hat{q}^{+} = q^{-1}\hat{q}^{-1} \tag{18}$$

† In [1], the authors adopted $\alpha = -1$ and insisted that $\int_{-\infty}^{\infty} f(x)Dx = \int_{-\infty}^{\infty} q^l\hat{q}^l f(x)Dx$ hold. However, equation (9) indicates that the above formula leads to a serious contradiction.

‡ $\alpha = -1$ is not included in this string.

we obtain

$$D^{+} = -q^{-\alpha-1}\hat{q}^{-\alpha-1}D. \tag{19}$$

Then we obtain the Hermitian momentum operator:

$$P = \mathrm{i}\hat{q}^{-(\alpha+1)/2}D. \tag{20}$$

We can easily check that

$$P^{+} = P. \tag{21}$$

Therefore, the correct $q$-deformed Schrödinger equation is given by

$$-\frac{1}{2m}\hat{q}^{-(\alpha+1)/2}D\hat{q}^{-(\alpha+1)/2}D\Psi(x) + \frac{(\alpha+1)}{2}kx^{2}\Psi(x) = E_{q}\Psi(x) \tag{22}$$

where $E_q$ denotes a $q$-deformed energy eigenvalue.

In this comment we pointed out a serious mistake in [1] and recommended the correct $q$-deformed one-dimensional Schrödinger equation.

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## References

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